Fundamentals of Magnetic Island Theory in Tokamaks

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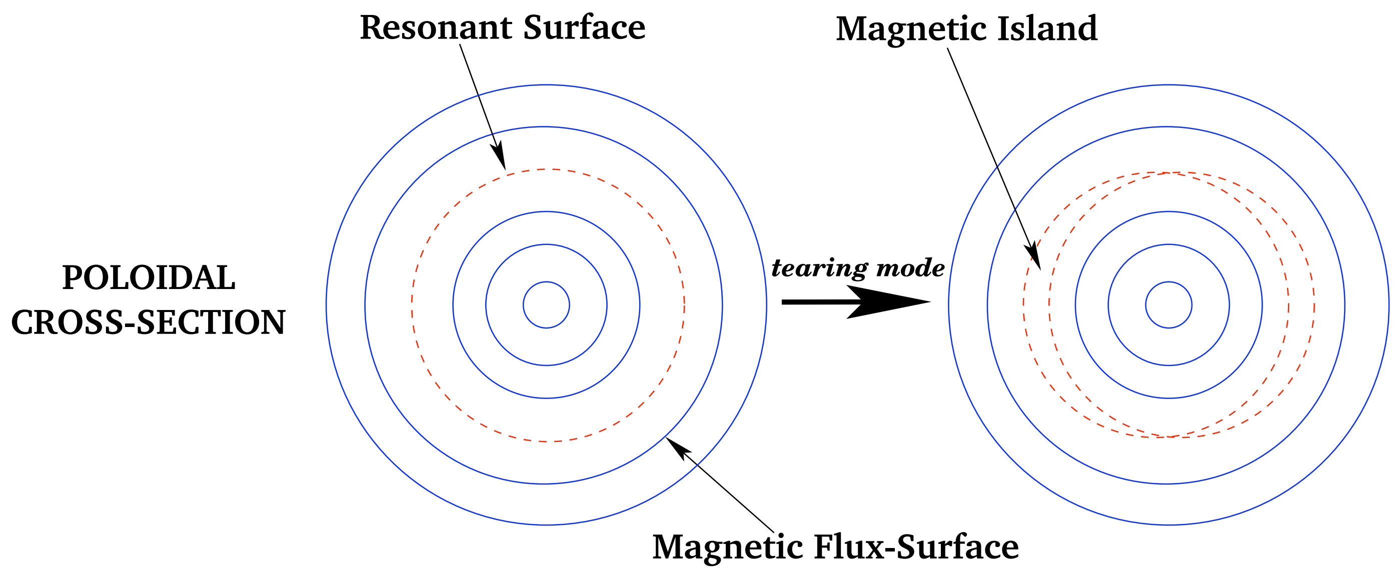
http://farside.ph.utexas.edu/talks/talks.html

Macroscopic Instabilities

- n typ s f c r s pc ns t s n t s[a]
  - C t s t r p c e n n rc n c t ng ns t s c h sr p t p s n f c r sc n s C n y t ng p s pr ss r n c rr n t
  - y gr ng t r ng ns t s c h rc n c t g n c s rfc s t f r *magnetic islands* t h r y gr ng t h rc n n n t pr p r t s c h h r r t

[a] *MHD Instabilities*, G. Bateman (MIT, 1978).

Magnetic Islands



- … *onal magne c u su faces* … $\vec{k}\cdot\vec{B}=0$ … $\vec{k}$ … $\vec{B}$ …
- …

Need for Magnetic Island Theory

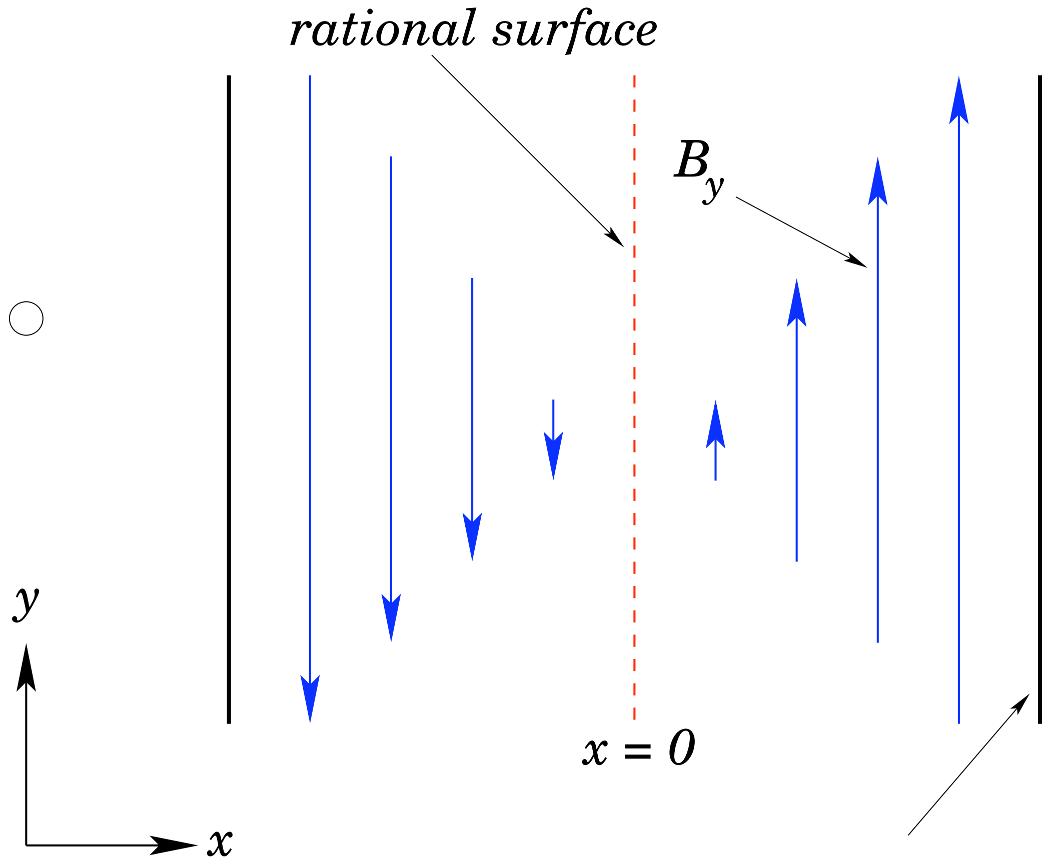
- [illegible]
- [illegible]
- [illegible]

MHD Theory

- [illegible] *fluid theory* [illegible]
- [illegible] *magnetohydrodynamical approximation* [a] [illegible] *single-fluid*
- [illegible] *slab approximation* [illegible]

[a] *Plasma Confinement*, R.D. Hazeltine, and J.D. Meiss (Dover, 2003).

Slab Approximation



Slab Model

- C r s rc r r s x y z L $\partial/\partial z \equiv 0$
- Ass pr s c f r r f r r $\vec{B}_z \vec{z}$
- A s r rg s r r B_z
- A rg s r r q r g c s r rg

$$L_s = B_z / B_y'(0).$$

- A s r r s r A f c c B_z
- P rfc r ryc rs $x = \pm a$
- r f r rg rs y $\vec{k} = (0, k, 0)$ s $\vec{k} \cdot \vec{B} = 0$ $x = 0$ c r s rfc $x = 0$

Outer Region

- n t r r g n c c pr s s s t f p s n n n r

Inner Region

- nn r r g n c n r n r n s rfc $x = 0$ Of n W □ 1 h r W s g n c s n h n x
- n nn r r g n n n c s n n n r c s n p s n r c n p r n
- nn r s n s sy p c y c h r s n r y n

Constant-ψ Approximation

- $\psi^{(1)}(x)$ g n r y s n t ry s g n c n y n x r n n r r g n

$$|\psi^{(1)}(W) - \psi^{(1)}(0)| \;\Box\; |\psi^{1}(0)|.$$

- *Cons an -ψ app o ma on* r $\psi^{(1)}(x)$ sc ns n n x

MHD Flow - II

- L t

$$M(\psi) = \frac{d\varphi}{d\psi}.$$

- s y s h n t h t

$$V_y = x M.$$

- y sy try $M(\psi)$ s *odd* f n t n f x n

$$M = 0$$

ns s p r tr /e n ns s p r tr n s n fr
P s *apped* t h n g n c s p r tr

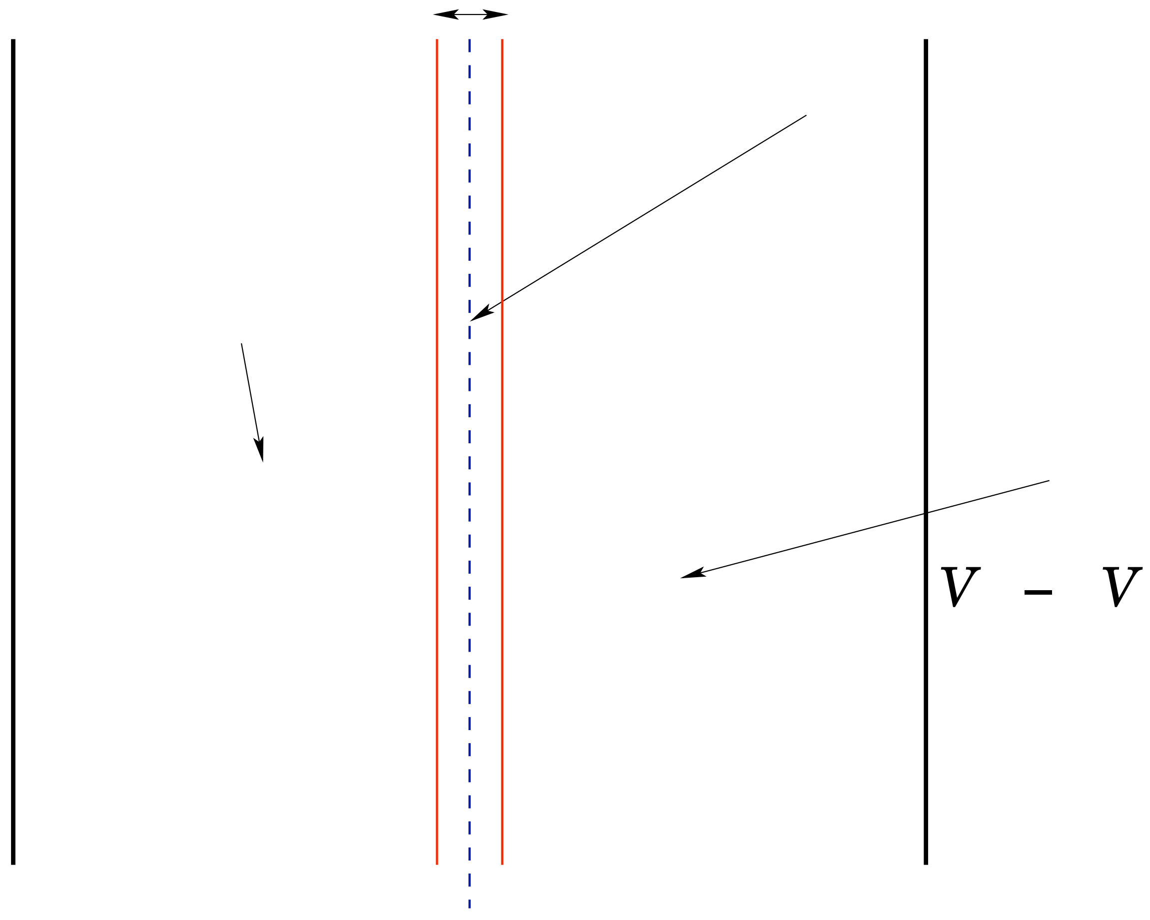
MHD Flow - IV

- ı ıhı

$$V_y = x\, M \longrightarrow |x|\, M_0$$

s $|x|/W \longrightarrow \infty$

- s h p α c ıy pr c h h ı n α s r h p s
- pc ı *isola ed* g n ıc s n α ı h *locali zed* c ıy pr
  gg sıs ı h ı $M_0 = 0$ f r s ı α s n α
- c n r n s n α fr *e* s n α pr p g ı s ı
  c


V – V

Rutherford Equation - I

- Asy pt tc ct h ng t n nn r n t r r g ns y s

$$\Delta'\psi = -4\int_{+\psi}^{-\infty} \langle J\cos\theta\rangle \, d\psi.$$

- n s n fr n s n f rt y q t n r c s t

$$[J,\psi] \square 0.$$

- n

$$J = J(\psi).$$

Rutherford Equation - II

- O h s

$$\frac{d\Psi}{dt} \cos$$

Rutherford Equation - IV

- g h r r ɑ r sy p ι t c c h ng ι n nn r n ɑ ι r r g ns y ɑ s [a]

$$\frac{0.823}{\eta}\,\frac{dW}{dt} \;\square\; \Delta' - 0.41\left(-\frac{d^4B_y^{(0)}/dx^4}{d^2B_y^{(0)}/dx^2}\right)_{x=0} W.$$

- n s ι r ι ɑ $dW/dt = 0$ s n ɑ ɑ ι h s

$W \approx$

MHD Theory: Summary

- r ng ɑ ns ɩ f $\Delta' > 0$
-

Drift-MHD Theory

- n o r f t ppr t n n ys s r t ns *c a ed pa cle d f eloc es* n oo t n t $\vec{E} \times \vec{B}$ c ty
- ss n y t o t h ry f p s
- C h rc t r s c ng t h s ρ s *on La mo ad us calcula ed n elec on empe a u e*
- C h rc t r s c c ty s o g n c c ty V_* h r

$$n\,e\,\vec{V}_* \times \vec{B}$$

Basic Assumptions

- R t ns σ f r s f s pc ty
- Ass p r c tr n h t tr nsp rt s c nt y str ng th t $T_e = T_e(\psi)$
- Ass $T_i / T_e = \tau = \text{constant}$ f r s f s pc ty

Basic Definitions

- r s
  - ψ g n c f n
  - J p r c rr n
  - φ g n c n r e s r f n
  - U p r n r c y
  - n c r n n r n s y n s n f r c gr n
  - V_z p r n c y
- P r rs
  - $\alpha = (L_n/L_s)^2$ h r L_n s q r n s y gr n s n g h
  - η r s s y D p rp n c r p r c s y $\mu_{i/e}$ p rp n c r n/ c r n s s y

Drift-MHD Equations - I

- [illegible]

$$
\begin{aligned}
\psi &= -x^2/2 + \Psi\cos\theta, \qquad U = \Box^2\varphi, \\
0 &= [\varphi - n, \psi] + \eta J, \\
0 &= [\varphi, U] - \frac{\tau}{2}\left\{\Box^2[\varphi, n] + [U, n] + [\Box^2 n, \varphi]\right\} \\
&\quad + [J, \psi] + \mu_i \Box^4(\varphi + \tau n) + \mu_e \Box^4(\varphi - n), \\
0 &= [\varphi, n] + [V_z + J, \psi] + D\,\Box^2 n, \\
0 &= [\varphi, V_z] + \alpha\,[
\end{aligned}
$$

Drift-MHD Equations - II

- y   try

Subsonic Islands [a]

- $$\varphi = \varphi(\psi), \quad n = n(\psi).$$

- $\varphi_e = \varphi - n$ … $\varphi_i = \varphi + \tau n$ …

Analysis - I

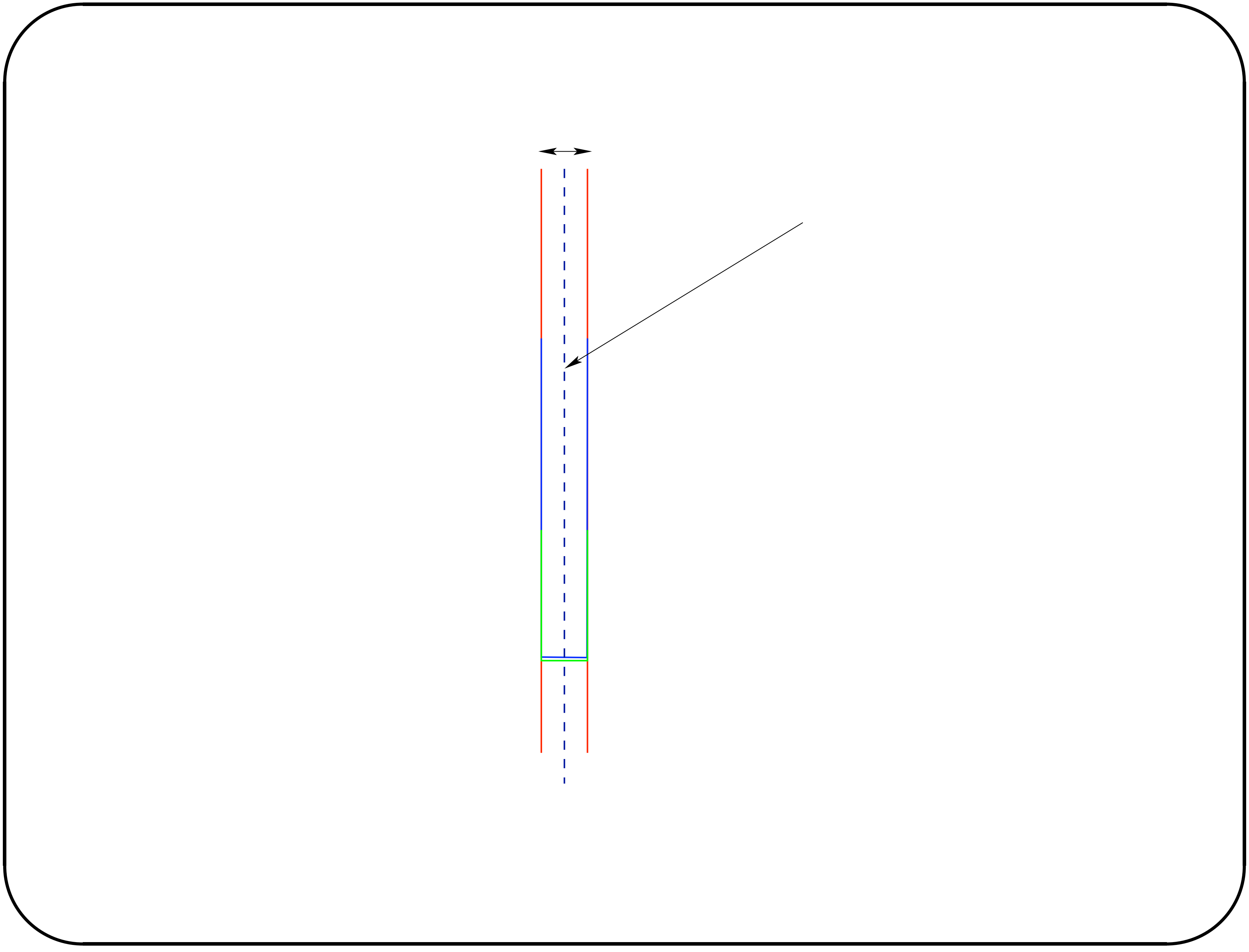
- ns ty q t n r c s t

$$0 \Box [V_z + J, \psi] + D \Box^2 n.$$

- rc ty q t n r c s t

$$0 \quad \Box \quad \left[-M U - (\tau/2)(L U + M \Box^2 n) + J, \psi\right]$$
$$+\mu_i \Box^4(\varphi + \tau n) + \mu_e \Box^4(\varphi - n).$$

- s rfc r g t h q t ns rc ng t h t $\langle[A, \psi]\rangle = 0$



Island Propagation

- As $|x|/W \to \infty$ pc ι $V_{y\,E\times B} \to V_{EB} - V$ h r V_{EB} s
  np rι r e n s n $\vec{E}\times\vec{B}$ c ιy ι r ι n s rfc n
  fr n V s s n p h s c ιy n fr

- n

$$V = V_{EB} + \frac{(\mu_i\,\tau - \mu_e)}{(1+\tau)\,(\mu_i + \mu_e)}.$$

ι np rι r n c ιr n c ι s n fr

$$V_i = V_{EB} + \tau/(1+\tau), \quad V_e = V_{EB} - 1/(1+\tau).$$

- n

$$V = \frac{\mu_i}{\mu_i+\mu_e}\,V_i + \frac{\mu_e}{\mu_i+\mu_e}\,V_e.$$

  s n p h s c ιy s *scos y e n ed a e a e* f
  np rι r n c ιr n c ι s

Polarization Term - I

- rc ty q t h y σs

$$J_c \square \frac{1}{2}\left(x^2 - \frac{\langle x^2\rangle}{\langle 1\rangle}\right)\frac{d[M(M+\tau L)]}{d\psi} + I(\psi)$$

tsσ s p r tr h r J_c s p rt f J t h $\cos\theta$ sy try

- As f r s rfc r g f O h s y σs

$$\langle J_c\rangle = I(\psi)\langle 1\rangle = \eta^{-1}\frac{d\Psi}{dt}\langle\cos\theta\rangle.$$

- σh

$$J_c \square \frac{1}{2}\left(x^2 - \frac{\langle x^2\rangle}{\langle 1\rangle}\right)\frac{d[M(M+\tau L)]}{d\psi} + \eta^{-1}\frac{d\Psi}{dt}\frac{\langle\cos\theta\rangle}{\langle 1\rangle}.$$

Polarization Term - II

- Asy pt tc ct h ng t n nn r n t r r g ns y s

$$\Delta'\Psi = -4\int_{+\Psi}^{-\infty} \langle J_c \cos\theta \rangle \, d\psi.$$

- t ng s rfc nt gr s ng s f pr s s t ns f r M n L

Drift-MHD Theory: Summary

- [illegible]
- [illegible]
- [illegible]
- [illegible]