Plasma Rotation in

Mechanisms have been developed to control/affect plasma rotation

- Toroidal rotation is influenced by:
 - External sources --- neutral beams
 - Intrinsic rotation --- topic of considerable research
 - A number of mechanisms have been proposed for intrinsic rotation --- turbulence, etc.

Describing toroidal rotation in tokamaks is a transport problem

- To date, most treatments describing toroidal rotation evolution rely on:
 - Braginskii

Thesis

Outline

Starting point for the calculation is the plasma kinetic equation

• Plasma kinetic equation for $f_s(x,v,t)$.

$$\frac{f_s}{t} + \frac{r}{v^{\#}}$$

- $C(f_s) = Fokker-Planck collision operator$
- $-S(f_s) = Kinetic source --- applied RF fields, neutral beams, etc.$
- Fluid moment equations are obtained from velocity-space moments of the plasma kinetic equation

$$d^{3}\vec{v}(1,m_{s}\vec{v},\frac{m_{s}v^{2}}{2})[\frac{\#f_{s}}{\#t}+\vec{v}~~\%f_{s}+\frac{q_{s}}{m_{s}}(\vec{E}+\vec{v}~\&~\vec{B})~~\frac{\#f_{s}}{\#\vec{v}}=C(f_{s})+S(f_{s})]$$

 Evolution equations for low order velocity space moments (n_s, V_s, p_s)

A number of assumptions are made to make analytic progress

- Small gyroradius expansion
 - Consequences for how we describe flows
 - Lowest order --- MHD force balance

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Small gyroradius expansion is used

• Gyroradius expansion: order terms and physical processes

Transport equations for density and pressure are obtained by flux surface averaging

• Flux surface averaging density and energy equations with V' = dV/d

$$\frac{n_{0s}}{t} + \frac{1}{V'} - (V')$$

 Cross-field particle/ heat fluxes due to collisional and fluctuation processes

The different orders of the momentum balance equation refer to different timescales

- To leading order in , MHD force balance
 - Summing radial momentum balance \vec{J}_0 $\vec{B}_0 = \# p_0$
 - Radial force balance produces relationship between toroidal, poloidal flows, E, and pressure gradient

$$\mathbf{0} = \stackrel{\mathbf{I}}{e} \begin{bmatrix} n_{i0}q_i(E + V_i & B) & p_i \end{bmatrix}$$

$$\Omega_t \quad V_i \qquad V \qquad = \quad \left(\frac{d}{d} + \frac{1}{n_i q_i} \frac{dp_i}{d} \quad qV \right)$$

$$V_t = \frac{E_r}{B_p} = \frac{1}{n_i q_i} \frac{dp_i}{dr} + \frac{B_t}{B_p} V_p$$

• First order flows are on magnetic surfaces $V_1 \sim$

$$\overset{\mathsf{\Gamma}}{V_1} = \overset{\mathsf{\Gamma}}{e} \overset{\mathsf{\Gamma}}{V} \# \qquad + \overset{\mathsf{\Gamma}}{e_{\mathscr{B}}} \overset{\mathsf{\Gamma}}{V} \# \qquad \mathscr{B} = V_{\parallel} \frac{\overset{\mathsf{L}}{B_0}}{B_0} + \overset{\mathsf{\Gamma}}{V_{\&}}$$

• Radial flows perpendicular to flux surfaces are second order

Poloidal flow is obtained from parallel force balance

Viscous damping occurs

After determining the poloidal flow, there is a unique relationship between E_r and the toroidal rotation

Recalling the radial force balance relationship

$$\Omega = \stackrel{\mathsf{f}}{V} \cdot \nabla \zeta = -(\frac{d\Phi}{d\psi} + \frac{1}{n_i q_i} \frac{dp_i}{d\psi} - q \stackrel{\mathsf{f}}{V} \cdot \nabla \theta)$$

Austin,

- $<\vec{B}_{0}$ # ;>%0 Using parallel momentum balance result —
- Relationship between radial electric field and toroidal flow

$$V_{t} = \frac{E_{r}}{B_{p}} \# \frac{1}{n_{i}q_{i}B_{p}} \frac{dp_{i}}{dr} + \frac{1.17}{q_{i}B_{p}} \frac{dT_{i}}{dr}$$
Pressure/temperature profiles determined by transport processes

Poloidal flow damping produces parallel plasma flow

$$\begin{split} & \prod_{i=1}^{r} \prod_{i=1}^{r}$$

Electron parallel momentum balance produces parallel Ohm's law

Following the same logic for the parallel electron momentum balance equation

$$= {}_{e}e < \vec{E} \# \vec{B} > < \vec{B} \# \# \vec{\aleph}_{e} > + < \vec{B} \# \vec{R}_{e} > + < \vec{B} \# \vec{S}_{e} >$$

$$_{e} _{e} < \vec{B} \ \# \ \vec{V}_{e} \# \ \vec{V}_{e} > _{e} \ e < \vec{B} \ \# \ \vec{V}_{e} \& \ \vec{B} >$$

- Using collision friction and neoclassical closure from kinetic theory $\begin{array}{c} r & r & r \\ B_0 & R_e \ \# & r \\ r & t \\ < B_0 \ \% \ \&_{e\parallel} > = m_e n_{e0} < B^2 > (\mu_{e00} U_e \ + \mu_{e01} Q_e \) \end{array}$

- Parallel Ohm's law

$$< E B_{0} >= \#_{\parallel}^{nc} < J B_{0} > \#_{\parallel}[< J B_{BS} > + < J B_{CD} > + < J B_{dn} >]$$

$$\begin{array}{c} \underset{\parallel}{\overset{nc}{\underset{\parallel}{n_{e}}}}{\overset{nc}{\underset{\parallel}{n_{e}}}} = \underset{\parallel}{\overset{m}{\underset{\parallel}{n_{e}}}} \left(1 + \frac{\#\mu_{e00}}{\#\mu_{e}} \right) \\ < J_{\%} \overset{r}{\underset{B_{CD}}{n_{e}}} > = \& \begin{array}{c} \overset{r}{\underset{\parallel}{n_{e}}} \overset{r}{\underset{\parallel}{n_{e}}} B_{BS} > = \& \frac{\#\mu_{e00}}{\#\mu_{e}} \left(I \frac{dp}{d} \& n_{e}eU_{i(} < B^{2} > \right) \\ & \\ = & J_{\%} \overset{r}{\underset{B_{CD}}{n_{e}}} > = \& \begin{array}{c} \overset{r}{\underset{\parallel}{n_{e}}} \overset{r}{\underset{\parallel}{n_{e}}} \left(I \frac{dp}{d} \& n_{e}eU_{i(} < B^{2} > \right) \\ & \\ \end{array} \right)$$



Parallel Ohm's law is used to describe collisional ambipolar particle flux

• Consider the particle fluxes from collisional friction

$$a_{s} = \frac{1}{q_{s}} < e \quad R_{s} > n_{0} < e \quad E > \qquad R_{e} \quad n_{e} e (\prod_{\parallel} J_{\parallel} + J_{\parallel}) = R_{e}$$

- Vector identity used to facilitate analysis
- Collisional-friction can be decomposed into parallel and perpendicular contributions

Plasma fluctuations influence particle flux/toroidal momentum balance

- At O(²), plasma fluctuation effects enter into the toroidal momentum balance
 - Microturbulence effects --- turbulent Reynolds/ Maxwell stresses
 - 3-D magnetic fields --- error fields, applied 3-D coils
 - Resonant magnetic perturbations ---> localized electromagnetic torques
 - Non-resonant magnetic perturbations ---> Neoclassical toroidal viscosity
- In general, these effects are not intrinsically ambipolar and hence will affect toroidal momentum balance.

3-D magnetic fields produce neoclassical toroidal viscous forces (NTV) throughout the plasma

- In an axisymmetric magnetic field, the toroidal component of the parallel viscous stress tensor is zero (µdB/ d = 0)
 - However, in the presence of 3-D magnetic fields, toroidal torques on toroidally flowing plasmas are generated.
 - Physics --- transit-time magnetic pumping, banana-drift, rippletrapping effects
 - Generally, the ion component dominates (the ion root of stellarator physics)
 - Ion viscous damping coefficient µ_{it} depends on collisionality, E_r
 - B_{eff}²

The NTV force is felt throughout the plasma

- Unlike torques due to resonant 3-D magnetic fields, the NTV force is global
 - Applied 3-D fields on NSTX demonstrated the damping effect of toroidal flow (Zhu et al, PRL '06)



- Favorable comparison to analytic predictions
- NTV physics has been seen on NSTX, DIII-D, MAST, JET

Experiments on DIII-D have demonstrated the presence of the NTV offset velocity

• Off-set rotation velocity observed on DIII-D (Garofalo et al '08)



Initially, slowly rotating Plasmas sped up to the Offset NTV velocity when 3-D fields are applied

Recent experiments on DIII-D have demonstrated a peak in the NTV force at zero radial electric field

- The toroidal damping rate (μ_{ti}) is sensitive to the value of the radial electric field
 - Damping rate corresponds to different collisionality regimes of stellarator neoclassical transport
 - Smoothed formula constructed to model different collisionality regimes (Cole et al, '10)
 Recent experiment on DIII-D

$$\mu_{ti} \sim \frac{n v_{ti}^2 \sqrt{\hat{\#}}}{\langle R^2 \rangle [0.3|_{|\%B|} \sqrt{\hat{\#}} + 0.04 \hat{v}^{3/2} + |_{E}|^{3/2}]}$$

Peaks at
$$_{\rm E} \sim 0$$

4th ITER International Summer School Austin, TX May 31, 2010 Recent experiment on DIII-D Demonstrates peak NTV at $_{E} \sim 0$



Particle flux has 8 non-ambipolar contributions

• Not intrinsically ambipolar

Zero radial current produces torque balance relation

• Summing radial species currents to obtain net radial plasma current

$$< J \cdot \nabla \psi > = \sum_{s} q_{s} \Gamma_{s} = \sum_{s} q_{s} \Gamma_{s}^{NA}$$

- Charge continuity requires no net radial current $\frac{\partial}{\partial t} < p_q$
- Setting radial current equal to zero produces a comprehensive toroidal torque balance relation

Toroidal rotation equation includes many different effects

• Equation for toroidal angular momentum density $L_t = m_i n_{i0} < R^2 \Omega_t >$

 $\frac{1}{V'} - (V'L) = \# \langle \vec{e} \rangle \langle \hat{k} \rangle |_{\parallel}^{NA} \rangle \# \langle \vec{e} \rangle \langle \hat{k} \rangle |_{\parallel}^{NA} \rangle \# \langle \vec{e} \rangle \langle \hat{k} \rangle |_{\parallel}^{NA} \rangle \# \langle \vec{e} \rangle \langle \hat{k} \rangle |_{\parallel}^{NA} \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle \langle \vec{f} \rangle \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle \langle \vec{f} \rangle \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle \langle \vec{f} \rangle \rangle + \langle \vec{e} \rangle \langle \hat{k} \rangle \langle \vec{f} \rangle \langle \vec{f} \rangle \langle \vec{f} \rangle \rangle \langle \vec{f} \rangle \rangle + \langle \vec{f} \rangle \rangle \langle \vec{f} \rangle \rangle + \langle \vec{f} \rangle \rangle \rangle + \langle \vec{f} \rangle \langle$

Collision damping

< **r**

 Microturbulence-induced ion Reynolds stresses causes radial transport of L_t (diffusion, pinch, residual stress)

Toroidal rotation determines radial electric field required for net ambipolar particle flux

- From toroidal rotation equation, radial electric field is determined
- The resultant electric field causes the electron and ion

Summary

- Comprehensive transport equations for n, T, Ω_t have been derived
- Radial, parallel and toroidal components of force balance are considered
 - Radial force balance --- relationship between V_t , V_p , E_r and dp_i/d
 - Parallel viscous damping determines neoclassical Ohm's law and poloidal ion flow
 - Radial particle fluxes arise from average toroidal torques on the plasma
- Radial particle flux has many contributions --- ambipolar and nonambipolar
- Requiring ambipolar particle flux yields evolution equation for toroidal angular momentum density

Summary

- 3-D magnetic fields have an important effect of flow evolution
 - Localized EM torques from resonant magnetic fields
 - Neoclassical toroidal viscosity (NTV) from variations in | B|
- Many aspects of NTV theory are being tested against experiments
 - Global damping of toroidal flow profile
 - Appearance of an offset rotation $\sim dT_i/d$
 - Peak of NTV torque near $E_r \sim 0$