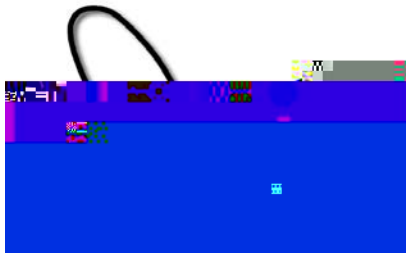


Signal processing techniques for characterizing nonlinear processes in turbulent plasmas

Gérard BONHOMME

Laboratoire de Physique des Milieux IonisØs et Applications, CNRS-UMR 7040, UniversitØ Henri PoincarØ Nancy

Gerard.Bonhomme@lpmi.uhp-nancy.fr

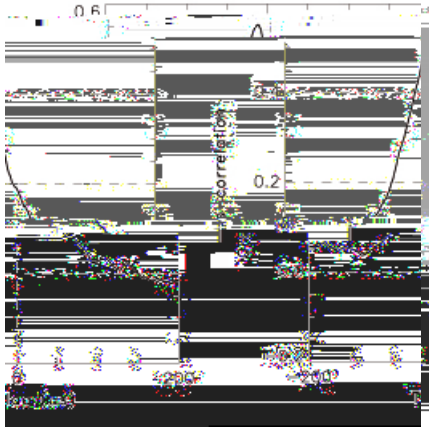
Thanks to: F. Brochard, E. Gravier, A. Scipioni, P. Rischette, ***LPMIA, Nancy***

T. Windisch, O. Grulke, T. Klinger, ***IPP-Greifswald***

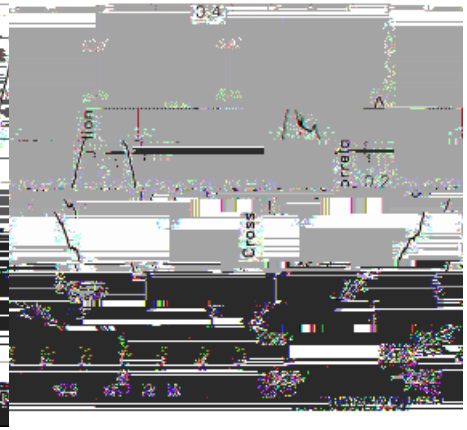
J. Gunn, N. Fedorczak, P. Devynck, ***DRFC-Association EURATOM-CEA Cadarache***

Tore-Supra Shot #35000

Analyzed signals: Isat_2A et Isat_2B
for ***r*** = 15, 20, 35, 70 mm (distance to LCFS)

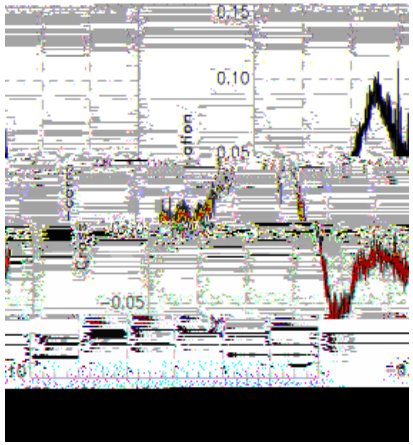


I1A-I2A



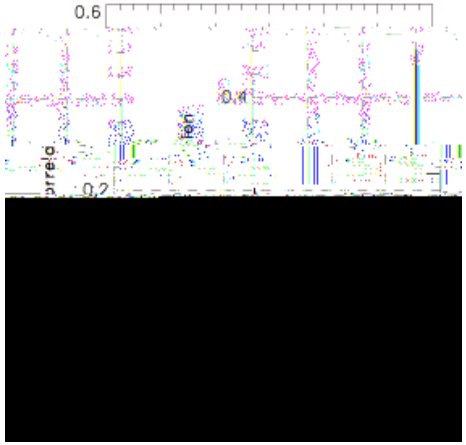
I2A-I3A

Cross-correlations r = 15 mm



I1A-I2A

I2A-I3A



I1A-I3A

Non Gaussian PDF related to intermittency

Cross-correlations r = 70 mm

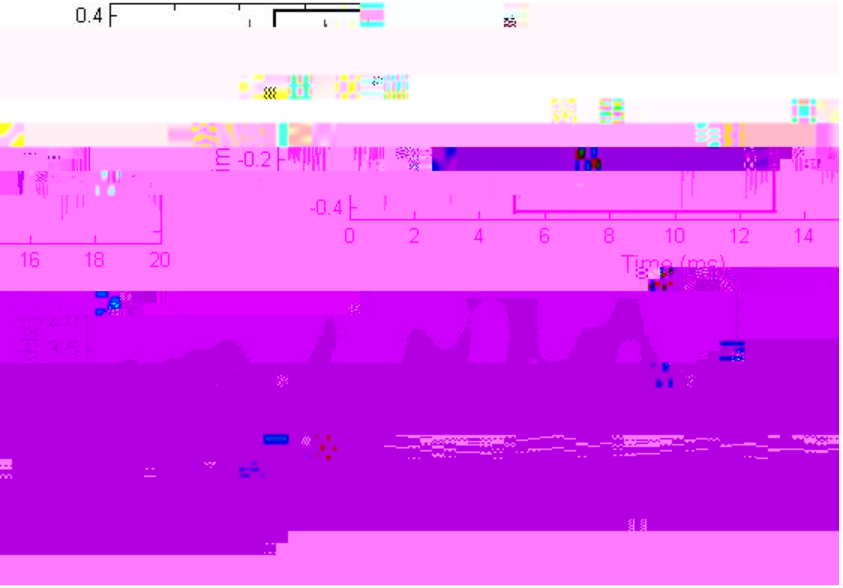
Tore-Supra Shot #35000

Drawbacks of Fourier methods

Fourier transform definition: $$F(\varpi) = \int_{-\infty}^{+\infty} f(t)\, e^{-2i\pi\varpi t}\, dt \quad \Leftrightarrow \quad f(t) = \int_{-\infty}^{+\infty} F(\varpi)\, e^{2i\pi\varpi t}\, d\varpi$$

- ♥ ***F*** (ϖ) complex ♥ Information on time localisation contained in the phase
  - ♥ difficult access
- Example 1 ⇓ A musician playing either successively two ∏ notes, or simultaneously these two notes ♥ same amplitude spectra

Periodic pulling



K-H instability

U_d=50 volts

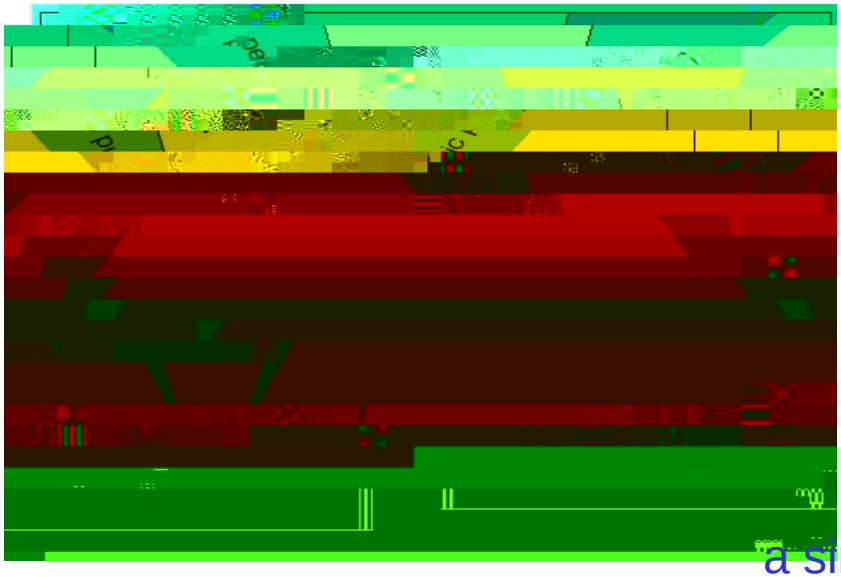
Time series "y5a02"

at r = 4 cm (in the shear layer)

A=1., κ =1.

ϖ_1 = 1.2

"VdP1.0-1.2"



a simple model: the forced van der Pol oscillator

Time-frequency analysis

- pulse

$e^{4\zeta(t4\vartheta)^{\cdot}}$ ɵ ϖ ƫ$_0$

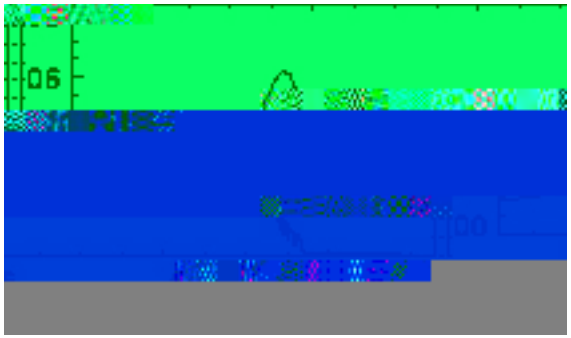
ζ = 0 ϑ = 1 ϖ_0 = 1 0

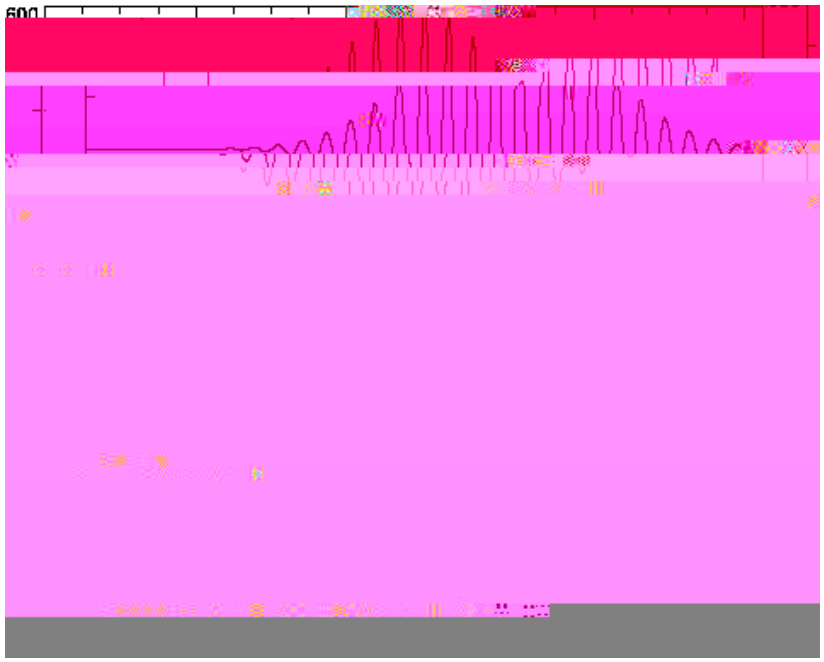
- chirp

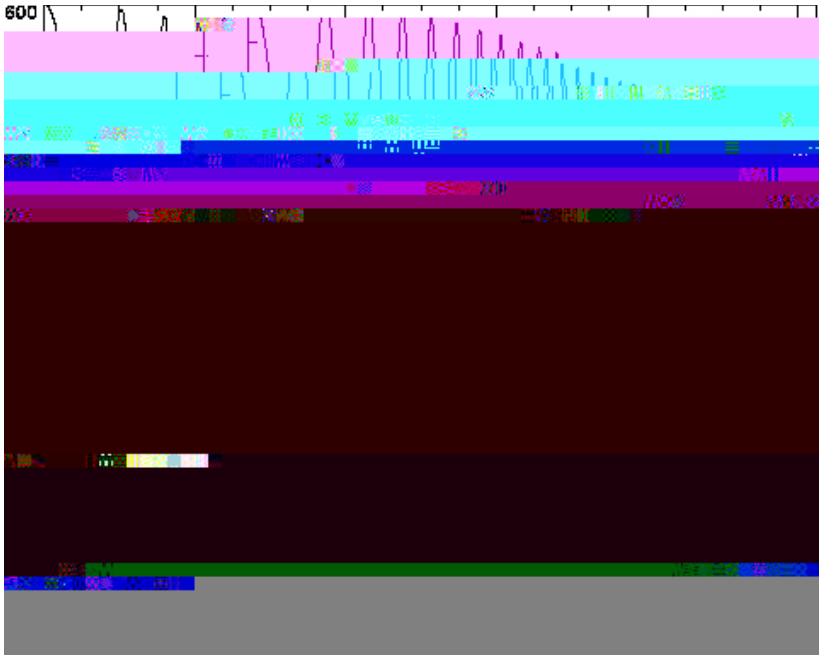
$e^{4\phi t^{\cdot} /\iota^{\cdot}}$ ɵ ⁞ ϕ(f ƫ$_0$2 $\eta t^{\cdot}$)

ι = 1 0 ;

η = 0 f_0 = . 0 .







Mallat tree (pyramid algorithm)

Solutions :

$$\begin{cases} c_0 = \frac{1}{4}(1+\sqrt{3}) \,, & c_1 = \frac{1}{4}(3+\sqrt{3}) \\ c_2 = \frac{1}{4}(3-\sqrt{3}) \,, & c_3 = \frac{1}{4}(1-\sqrt{3}) \end{cases}$$

$$f(t) = a_0 \Phi(t) + a_1 W(t) + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j - 1} a_{2^j + k} W(2^j t - k)$$

$$= a_0 \Phi(t) + a_1 W(t) + a_2 W(2t) + a_3 W(2t-1) + \sum_{j=2}^{\infty} \sum_{k=0}^{2^j - 1} a_{2^j + k} W(2^j t - k)$$

$$= a_0 \Phi(t) + a_1 W(t) + a_2 W(2t) + a_3 W(2t-1) + a_4 W(4t) + a_5 W(4t-1) + a_6 W(4t-2) + a_7 W(4t-3) + \sum_{j=3}^{\infty} \sum_{k=0}^{2^j - 1} a_{2^j + k} W(2^j t - k)$$

D4

D10

Practical considerations (1)

- The Continuous Wavelet Transform (Morlet):

  FFT computation of

  Practically $f=\{f_n\}$ et $N=2^m$

♦ ♦

♥ Oversampling of $F(\varpi)$ required

Practical considerations (2)

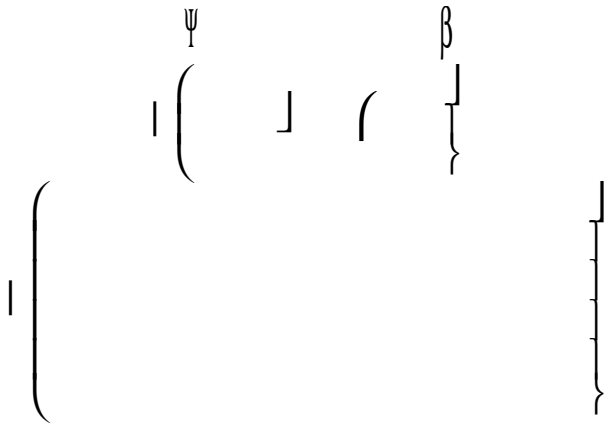
Discrete Wavelet Transform: ⇓ pyramid algorithm (no need for the $W(t)$)

example : $f=f(1:8) \xrightarrow{\frac{1}{2}L_3} f'(1:4) \xrightarrow{\frac{1}{2}H_2} f'(1:2) \xrightarrow{\frac{1}{2}H_1} f'(1)$

$\sharp\Leftrightarrow \frac{1}{2}H_3$ $\Leftrightarrow \frac{1}{2}H_2$ $\sharp\Leftrightarrow \frac{1}{2}H_1$ $\sharp\Leftrightarrow$

$a[5:8]$ $a[3:4]$ $a(2)$ $a(1)$

H_n et L_n are matrices build directly from the c_k coefficients (cf. Newland [1])



Times-series and self-similarity properties

Scale Invariance $\Downarrow$ self-similar stochastic process

- Definition and properties:
- Power spectrum ν characteristic exponent
- A

R/S analysis and the Hurst exponent (1)

The rescaled ranged statistics (R/S) method was proposed to evaluate the Hurst exponen.5 -1.20(()-3(R)-1(/)Tw 1-0.-80(()-3(R)-1(/)Tw 1-0.-80 27.96 95nce,the Hurs

Wavelets are self-similar by nature

Mother-wavelet $\pi(t)$ ♥ $\pi_{T,t_0}(t) = \pi\left(\frac{t - t_0}{T}\right)$ translation + dilatation

- The **wavelet variance** is a very useful alternative to spectral density function, and R/S analysis

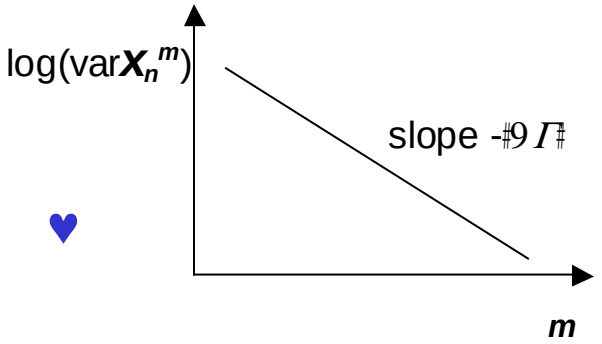
- Discrete wavelet transform

  octave scaling ⇓ $T_m = 2^{m-1}$ ♥ $\nu_X(2^j) \sum \text{var}\{W_j\}$ with $\vartheta_j = 2^{j-1}$

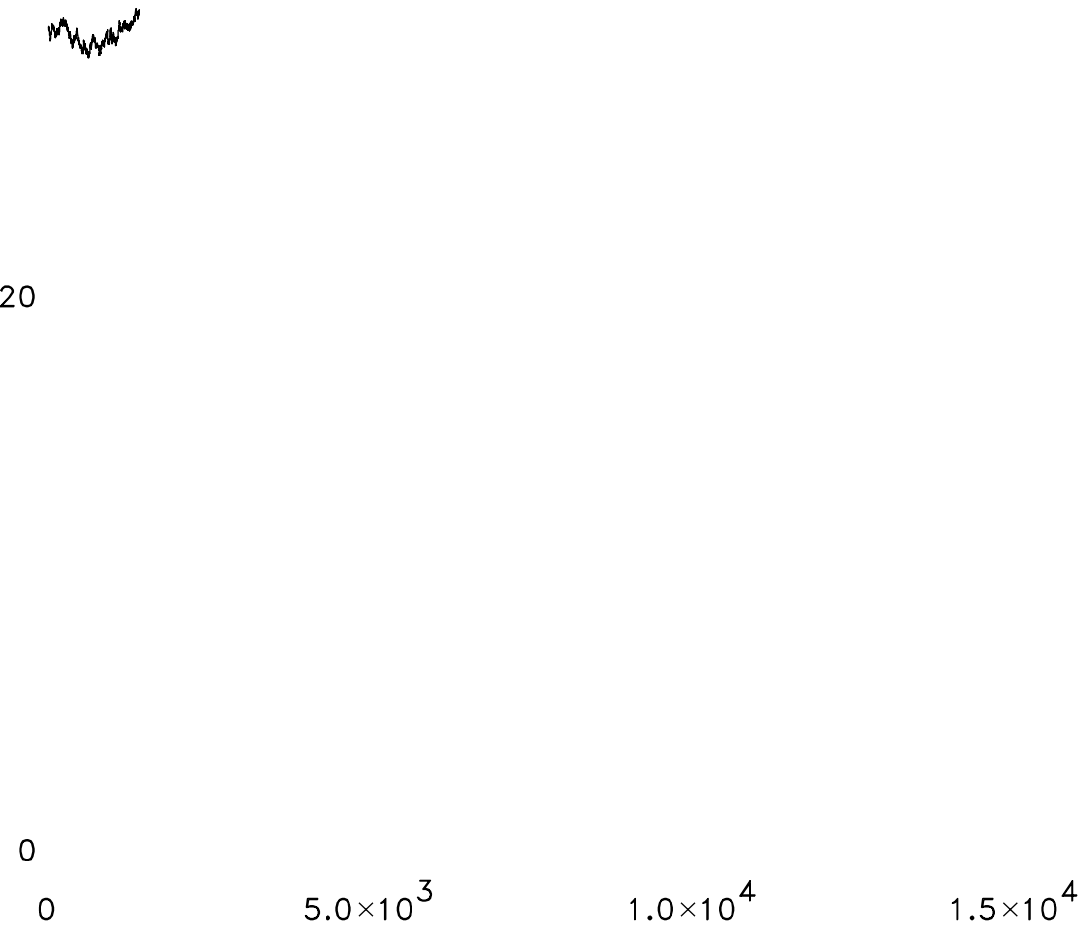
$$\sum_{j=1} \tau_X(\vartheta_j) \sum \omega_X$$

$$\tau_X(\vartheta_j) - \int_{1/2^j}^{1/2^{j+1}} S_X(f)\,df$$

♥ (if $S_X(f) \propto |f|^{\zeta}$) $\nu_X(2^j) \propto 2^{j\,1.41}$ ♥



Wavelet variance (DWT, Daubechies D20)



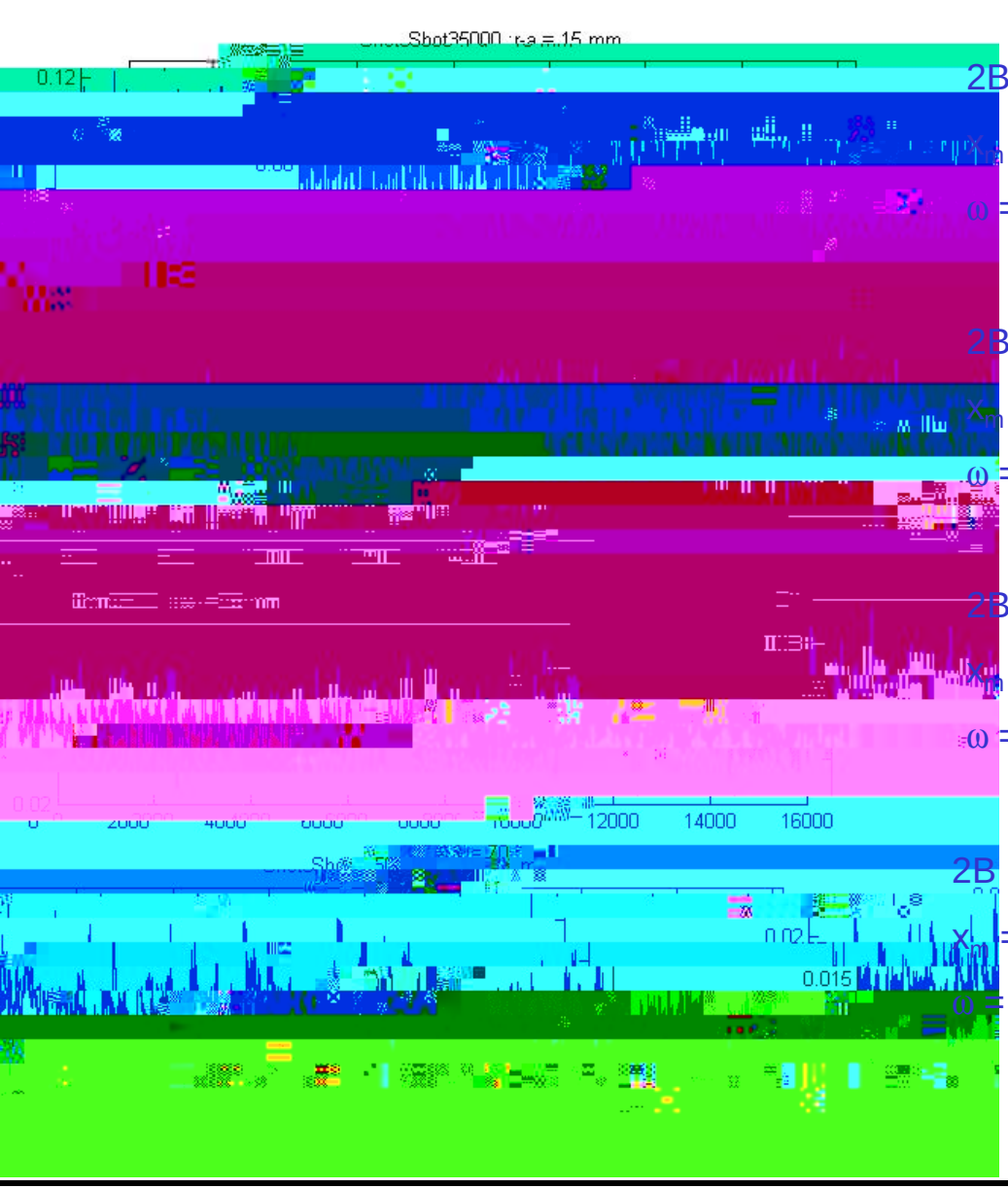
SOL turbulence (Tore-Supra data)

Tore - Supra: shot #22253

Two distinct behaviors can be seen on the spectrum:

- before breakpoint (BF) signal ~ fGn
- After breakpoint (HF) signal ~ fBm

Tore-Supra (Shot #35000)



2B Isat,

χ_m = 0,0822

ω = 0,0118

ν=1.84

2B Isat,

χ_m = 0,072

ω = 0,0094

ν=1.78

2B Isat,

χ_m = 0,0551

ω = 0,0076

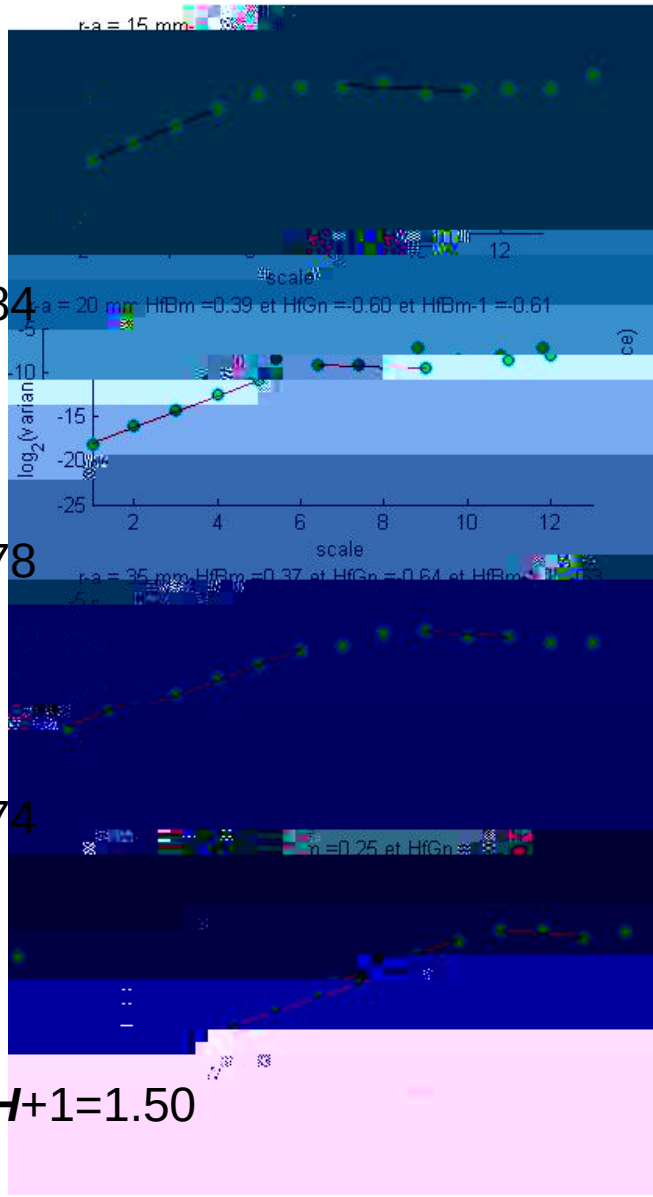
ν=1.74

2B Isat,

χ_m = 0,0144

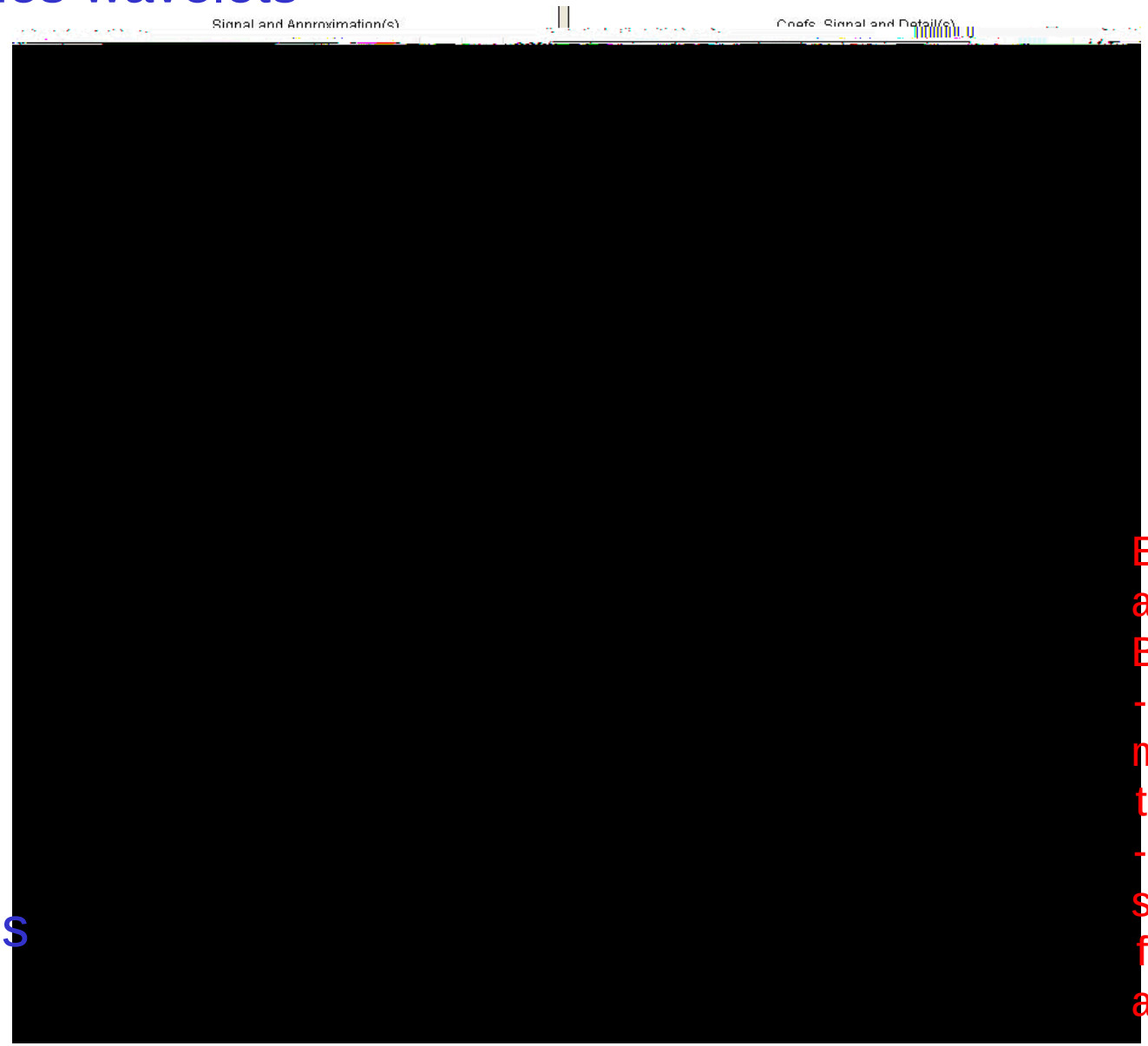
ω = 0,0023

ν=2***H***+1=1.50



Discrete wavelets: Analysis and reconstruction

Daubechies wavelets



Analysis

Synthesis

Efficient algorithmn,
But:
- physical meaning of the filtering?
- not well suited to time frequency analysis

Hilbert Transform and instantaneous frequency

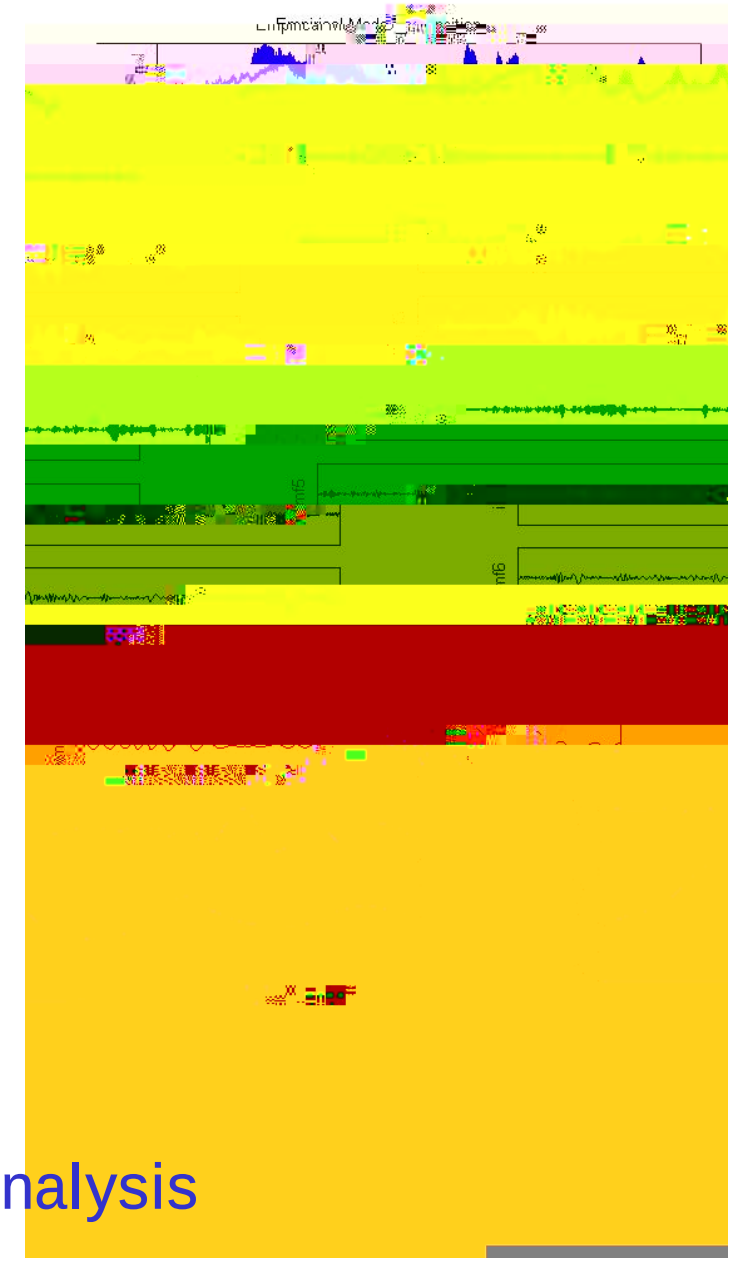
$$H[x(t)] \equiv \frac{1}{\pi} p.v. \int_{-\infty}^{+\infty} \frac{x(u)}{(t-u)} du$$

IMF = Intrinsic Mode Functions

1. **Initialize : $r_0(t) = X(t)$, j=1**
2. **Extract the j-th IMF:**
   - **a) Initialize $h_0(t) = r_j(t)$, k=1**
   - **b8.15 14.04 7.2**

$$X(t) \; | \sum_{j|1}^{n} imf_j(t) \; 2\, r_n(t)$$

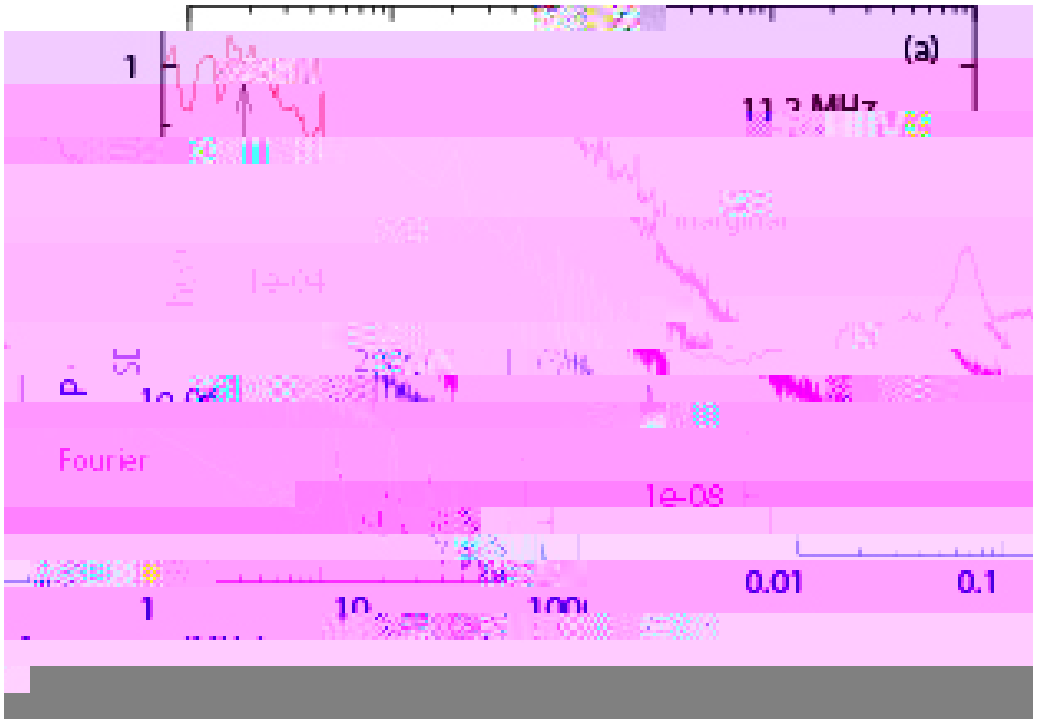
Analysis and Reconstruction (Plasma thruster data)



Analysis

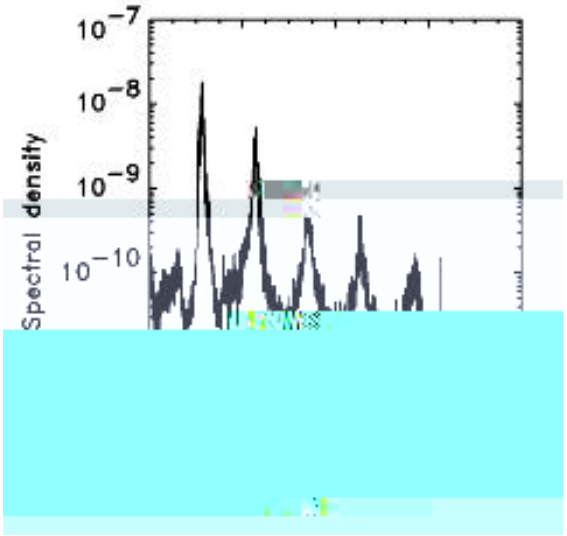
Synthesis

Marginal Hilbert spectrum vs Fourier spectrum



Because of the strong nonlinearity of HF oscillations the Fourier spectrum exhibits many peaks

All these peaks do not correspond to actual modes



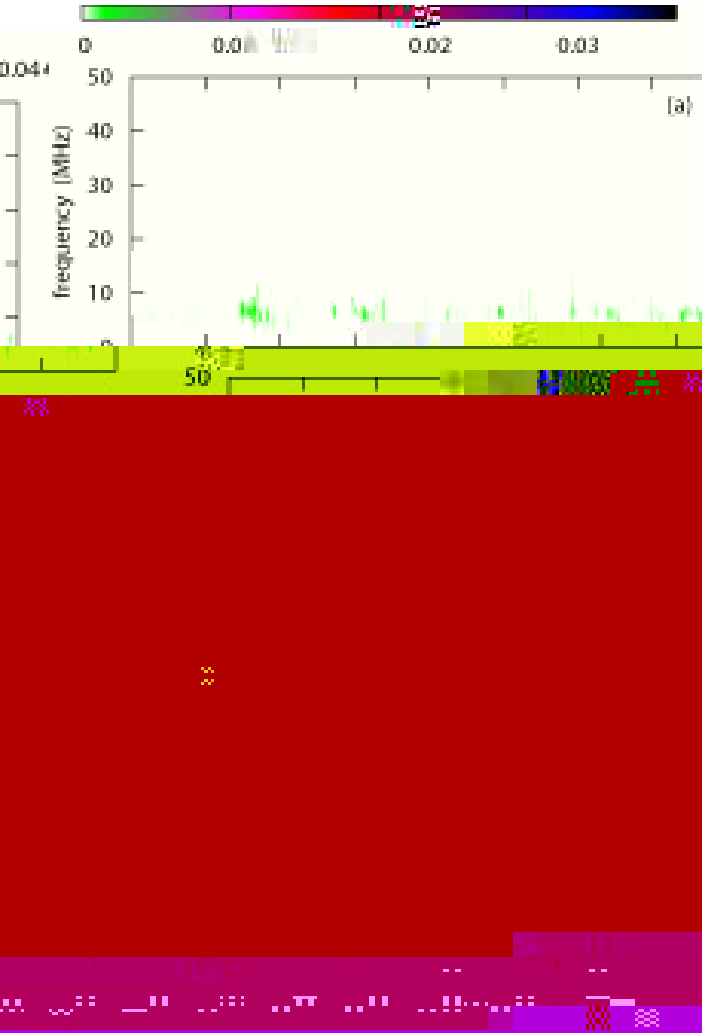
A peak in the marginal Hilbert spectrum corresponds to a whole oscillation around zero

Application to experimental time-series (Plasma Thruster)

IMF 1-4

IMF 9

Comparison with wavelet time-frequency analysis



Morlet ⇓ freq. = 375/scale ♥ 33 ⇐ 11.4 MHz

The Hilbert-Huang Transform method:

- Has proven to be a promising and attractive method to analyze non stationary and nonlinear time-series because of:

  - a very efficient ability in filtering different physical phenomena

  - accurate time-frequency

Joint wavelet phase-frequency spectra

- The Joint wavelet phase-frequency spectrum S (λ, ϖ) is obtained by calculating the quantity:

,(, 2 2

for a number of values of a and ϑ, with fixed $\div \vartheta$

$\varpi = 2\phi/a$ and λ phase of c $\Downarrow$ plot in the (λ, ϖ)-plane

♥ insight into the frequency-dependent phase relations that may exist between f and g (usually two spatially separated measurements of the same quantity)

moreover (if homogeneous turbulence)

$\Downarrow$ related to the dispersion relation $\varpi(k)$ for the process driving the turbulence

Non linear spectral analysis tools

Non linearity requires proper spectral analysis tools :

The Fourier method is based on the third-order spectrum

where $\varpi = \varpi_1 + \varpi_2$ and $\langle \cdot \rangle$ = ensemble average

- Wavelet cross bispectrum

- Wavelet cross bicoherence (normalized squared cross bispectrum)

Bicoherence as a Fourier tool

From Van Milligen, Wavelets in Physics, edited by J. C. Van Den Berg,
(Cambridge University Press, 1999)

Bicoherence

Summed Bicoherence

allows to determine the coupling direction

Total bicoherence

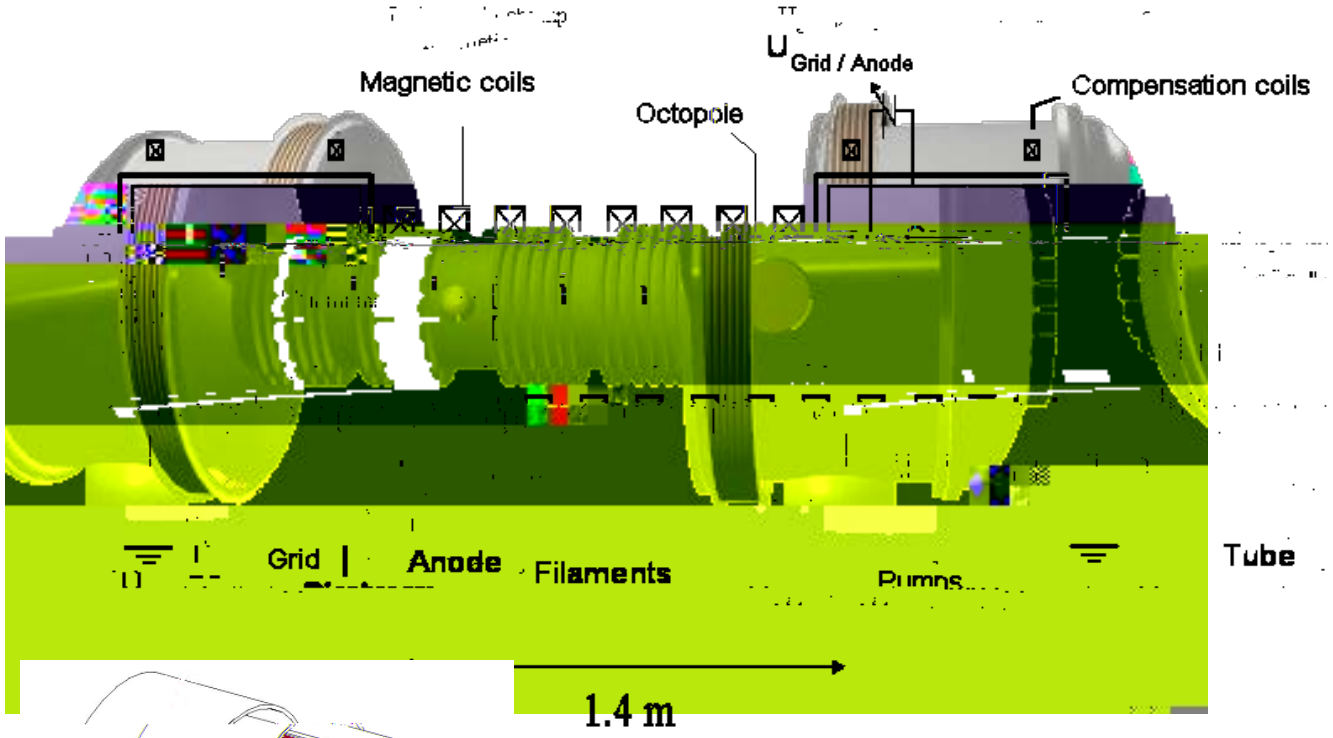
gives an indication on the amount of nonlinear coupling

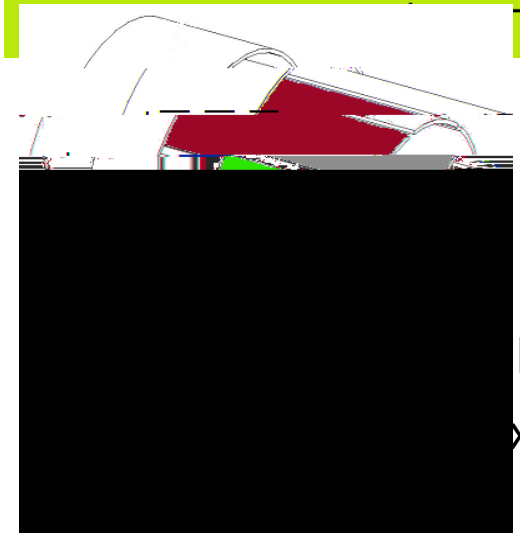
Staistical noise:

=> depends on the scale 1/ k

Results : dynamical analysis

Experimental Setup: the MIRABELLE device





plates

xciter

Plasma typical parameters:

- Argon plasma
- B = 0 – 130 mT
- P ~ 0.5-5 E^{-4} torrs
- Te ~ 1 – 3 eV , Ti ~ 0.02 eV
- n_e ~ 10^{15} – 10^{16} m^{-3}
- ψ_S ~ 0.5 – 3 cm

Spatiotemporal control of a weak turbulent state

The exciter plates are localised outside of the plasma ♥ no limiter effect.

In Mirabelle, it is possible to select between "flute" modes (Kelvin-Helmholtz, Rayleigh-

Spatiotemporal control of a weak turbulent state

Example 1 : forcing of a m= 1 Kelvin-Helmholtz mode

A m=1 mode at F_{ex}=4kHz is applied to a ~3 à 7 kHz irregular mode, for two rotation directions (amplitude 2V)

Filtering out the conter - rotation by using a 1 st order band-pass filter

Spatiotemporal control of a weak turbulent state

Spatiotemporal effect

the m= 3 mode is not totally suppressed.

$k_{//} = 0$ before and after control: same kind of mode.

in the counter-rotation case, the applied spatiotemporal structure is simply superimposed. No coupling.

Bicoherencein-

Driving a Turbulent state

Fast camera data (test)

Raw data taken in the Mirabelle device

Drift wave, regular mode

After substracting mean value at each pixel

After wavelet processing

Conclusions and Perspectives

- Several methods are available
- The choice of the best method depends on the kind of information we want to extract from the data
- Fourier methods have many drawbacks when applied to non stationnary nonlinear signals
- Wavelets based tools are very useful, and especially to measure self-similiarity properties
- Among the last introduced methods the Hilbert-Huang transform has proven to be very efficient in filtering