Models for Global Plasma Dynamics

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Outline

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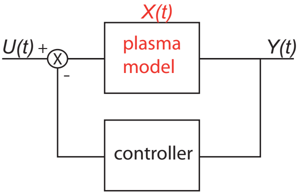
1. Models for Long-Wavelength Plasma Dynamics
   - Introduction
   - Fluid models: MHD and Hall-MHD
   - Gyrokinetic theory
2. Hamiltonian Gyrofluid Reconnection
   - Motivation
   - HEMGF: A Hamiltonian Gyrofluid Model
   - Reconnection

Relationship to control design process



$$\dot{X}(t) = f(X(t), U(t), t);$$
$$Y(t) = g(X(t), U(t), t).$$

- We focus on first two elements of control design process
  1. Make system model
  2. Verify model predicts behavior of system
  3. Design controller
  4. Test models in closed-loop simulation
  5. Implement and test implementation
  6. Deploy in operation
- We are concerned in this and subsequent talks with formulating and solving the equation for $X(t)$.

Linear analysis of dynamics

- Small motions away from the equilibrium state, $X = X_0 + x$ can be described by Taylor expansion of the force:

$$\underline{x} = \mathbf{M}x + O(x^2);$$

where **M** is the Jacobian operator (matrix):

$$\mathbf{M} = @F(X;U;t)=@X j_{X=X_0} \quad j \quad o;0104n6s0.24S41\text{-}1$$

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Electron momentum conservation determines the electric field

- Using $\mathbf{V}_e = \mathbf{V} \quad \mathbf{J}{=}ne$ in the magnetic force, the electron momentum conservation takes the form

$$m_e n \frac{d\mathbf{V}_e}{dt} = ne(\mathbf{E} + \mathbf{V} \quad \mathbf{B} \quad \mathbf{J}) \quad r\, p_e \quad r \quad e \quad \mathbf{J} \quad \mathbf{B}$$

- The only term that can balance the magnetic force is thus the electric force:

$$\mathbf{E} + \mathbf{V} \quad \mathbf{B} = 0:$$

Digression: Single vs. Two-fluid models

- MHD is sometimes called a "single fluid" model. But plasma conducts electricity: $\mathbf{J} = ne(\mathbf{V}_i \quad \mathbf{V}_e) \neq 0$.
- It is easy to see that

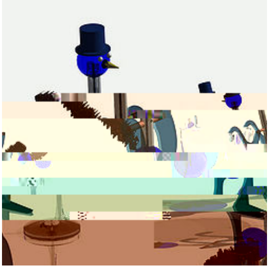
$$\frac{V_i \quad V_e}{V_i} \quad \frac{J}{neV_A} \quad \frac{d_i}{a} = \quad {}_i^{\,1/2} \quad 1;$$

where $\quad = \quad {}_i = a$.

- So MHD is a "quasi-single-fluid" in the same sense that it is a "quasi-neutral" theory:

$$\frac{n_i \quad n_e}{n_e} = \frac{{}_0\, r \quad \mathbf{E}}{n_e e} = \frac{V_A}{c}^{\;2} \quad {}_i^{\,1/2} \quad 1:$$

Fusion MHD events are sub-Alfvénic



- Typical times scales for sawtooth crash, ELM, disruption 100 s $_A$ 1 s.
- In the drinking bird toy, evaporation draws fluid up the tube, creating an inverted pendulum (c.f. D. Humphreys lecture)
- The evaporation rate , so the dip is preceded by a precursor oscillation.
- Fusion plasmas do not fit the “drinking bird” paradigm: instabilities
  1. often lack a discernible precursor.

Sub-Alfvenic MHD motion exhibits spatial resonances

- For $V \quad V_A$, the MHD equilibrium equation applies

$$\mathbf{J} \quad \mathbf{B} = \; r\,p:$$

- This determines $\mathbf{J}_? = (\mathbf{B} \quad r\,p)=B^2$.
- Ampere's law $r \quad \mathbf{J} = 0$ determines J_k:

$$\mathbf{B} \quad r\,(J_k=B) = \quad r \quad [(\mathbf{B} \quad r\,p)=B^2]:$$

- The operator $\mathbf{B} \quad r = (m \quad nq)B=Rq$ is singular on magnetic surfaces with closed field lines.

Sub-Alfvénic motions are described by the drift ordering

- Recall the electron momentum conservation

$$m_e n \frac{d\mathbf{V}_e}{dt} = ne(\mathbf{E} + \mathbf{V} \quad \mathbf{B} \quad \mathbf{J}) \quad r\, p_e \quad r \quad {}_e \quad \mathbf{J} \quad \mathbf{B}$$

- Assume $\mathbf{V} \quad \mathbf{B} \quad r\, p_e = ne$ This is the *drift ordering*.
- Eliminate **E** between the electron momentum conservation and Faraday's law,

$$\frac{@\mathbf{B}}{@t} = \quad r \quad \mathbf{E}:$$

Hall Magnetohydrodynamics

- There follows

$$\frac{\partial \hat{\mathbf{B}}}{\partial t} = -\nabla \times \hat{\mathbf{E}},$$

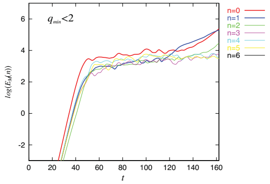
where

$$\hat{\mathbf{E}} = -\mathbf{V} \times \mathbf{B} + \mathbf{J} \times \hat{\mathbf{B}}/ne - (\nabla p_e + \nabla \cdot \Pi_e)/ne,$$

$$\hat{\mathbf{B}} = (1 - d_e^2 \nabla^2)\mathbf{B},$$

$$\mathbf{J} = \nabla \times \mathbf{B}/\mu_0.$$

The perils of two-fluid models



1. Hall MHD describes whistler waves and the ion cyclotron resonance. This is the curse of the drift ordering: to describe slow evolution we have to include fast waves!

$$! = kV_A \quad ! \quad ! = k^2 \; {}_s V_A$$

2. By including drift motion, two-fluid models enable all the drift instabilities!
3. The model is still missing important kinetic physics (FLR, parallel dynamics, etc. . .)
4. It is hard to parallelize.

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GFM II. Gyro-averaging

Dorland and Hammett defined the operator $_0^{1/2}$ as follows

$$_0^{1/2} = \exp(\tfrac{1}{2} \; r_?^2) I_0^{1/2}(\quad r_?^2) \; ; \qquad (3)$$

where $= T_s = T_{ref}$ and I_0 is the modified Bessel function of the first kind. The definition in Eq. (3) should be interpreted in terms of its series expansion

$$_0^{1/2} = 1 + \sum_{n=1}^{\infty} a_n(\; r_?^2)^n = 1 + (\; =2) \, r^2 + \quad ;$$

where the a_n are real numbers.

Hamiltonian versions of the GFM model can be constructed

- Why Hamiltonian? Recall that in MHD the equilibrium and charge conservation equations require that

$$\mathbf{B} \quad r\,(J_k{=}B) = \quad r \quad (\mathbf{B} \quad r\,P{=}B^2):$$

- Integrability of the charge conservation condition requires that

$$\text{I} \; \frac{d`}{B} \; r \quad \mathbf{J}_? = 0:$$

- Similar constraints must be satisfied by the vorticity, density, etc. . . When are these conditions satisfied?

Hamiltonians at work

- The ideal part of the fluid equations must be expressible in terms of Poisson brackets:

$$\partial_t \psi_j = \{\psi_j, H\} + D\nabla^2\psi_j.$$

- Poisson brackets generally possess families of geometrical invariants C_k called Casimirs.

$$\{\psi_j, C_k\} = 0.$$

- Equilibria are the extrema of the functional $F = H + \sum_k C_k$,

$$\delta F = 0.$$

The Hamiltonian formulation thus guarantees equilibrium

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Hamiltonian formulation for Alfvén dynamics

- Consider the following model that describes kinetic and inertial-Alfvén waves:

$$\frac{\partial n_i}{\partial t} + [\ ; n_i] = 0; \qquad (4)$$

$$\frac{\partial n_e}{\partial t} + [\ ; n_e] \quad c_A^2\, r_k J = 0; \qquad (5)$$

$$\frac{\partial}{\partial t}(\quad d_e^2 J) + [\ ; \quad d_e^2 J] + \ r_k n_e = 0; \qquad (6)$$

where

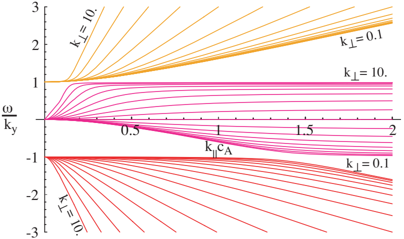
$$[f; g] = \mathbf{B}_0 \ \ (r f \quad r g)/B_0.$$

- The conserved energy is

$$H = \tfrac{1}{2} h c_A^2 (j r \quad j^2 + d_e^2 J^2) + n_e^2 + \quad n_i \qquad n_e i.$$

Dispersion relation

- The gyrofluid equation reproduces *exactly* the kinetic dispersion relation in both the kinetic-Alfvén and inertial regimes.
- Unlike FLR (Braginskii) models, it reproduces the band gap between the ion and electric drift frequencies.



Normal fields

- The Casimirs suggest the use of the normal fields n_i and $G \quad = \qquad d_e^2\, r^{\,2} \qquad {}_e n_e$.
- The equations of motion for the normal fields are

$$\frac{\partial n_i}{\partial t} + [\quad ; n_i] = 0; \qquad (7)$$

$$\frac{\partial G}{\partial t} + [\quad ; G \quad] = 0; \qquad (8)$$

$$\qquad (9)$$

where

$$= \qquad = {}_e$$

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IFS

Role of ion temperature in reconnection

- For constant-density, the model reduces to two fields:

$$\frac{\partial G}{\partial t} + [\quad ; G \quad] = 0;$$

where $G \quad = \qquad d_e^2 \nabla^2$

Gyrofluid reconnection

- For hot ions, by contrast,

(1

Summary (I)

- MHD is inadequate to model most MHD events in tokamaks.
- Two-fluid models provide a better description, but they still neglect important effects such as parallel heat flow, Landau damping, and finite Larmor radius.
- Two-fluid models unleash onto MHD the pandora's box of drift-acoustic turbulence: ITG, ETG, etc. . . Subgrid models may need to be developed.

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Summary (II)

- Gyrokinetic theory is free from the deficiencies of fluid models. Two of its own deficiencies are subjects of ongoing research:
  1. Quasi-static coupling to the compressional Alfvén wave;
  2. Collision operators
- The gyrofluid closure method holds considerable promise for understanding the role of FLR in MHD.
- The model that will enable us to understand ITER MHD has yet to be invented, perhaps by one of you.